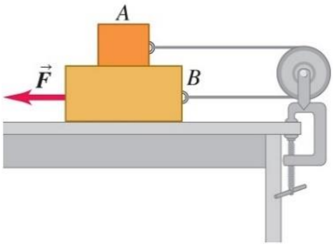
PHYS 101 – General Physics I Midterm Exam 2

Duration: 120 minutes Saturday, 26 November 2022; 09:30

1 . Block A in the figure has mass m_1, and block B has mass m_2. The coefficient of kinetic friction between all surfaces is μ_k. Blocks A and B are connected by a light, flexible cord passing around a fixed, massless, frictionless pulley. The gravitational acceleration is g.

(a) (5 Pts.) Draw a free body diagram for both blocks.

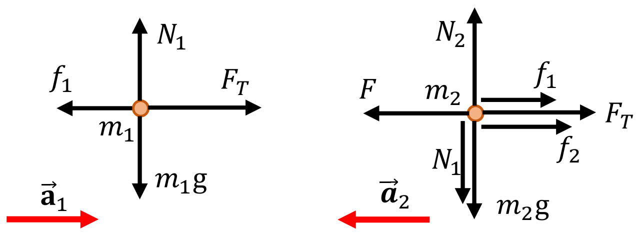


(b) (8 Pts.) Find the magnitude of the horizontal force $\vec{\mathbf{F}}$ necessary to drag block B to the left at constant speed.

(c) (12 Pts.) Find the tension in the cord and the magnitude of the force $\vec{\mathbf{F}}$ if it is large enough to accelerate the bottom block with acceleration a. (Express your result in terms of m_1, m_2, g, a, and μ_k.

Solution:

(a)



(b) In this case, we have $\vec{\mathbf{a}}_1 = 0$, $\vec{\mathbf{a}}_2 = 0$.

$$F_T - f_1 = 0\,, \qquad f_1 = \mu_k N_1 = \mu_k m_1 g \quad \rightarrow \quad F_T = \mu_k m_1 g\,.$$

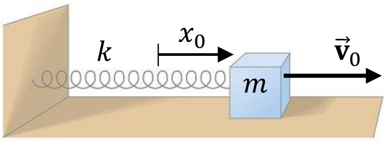
$$F - F_T - f_1 - f_2 = 0\,, \qquad f_2 = \mu_k N_2 = \mu_k (m_1 + m_2) g \quad \rightarrow \quad F = \mu_k (3m_1 + m_2) g\,.$$

(c)) In this case $|\vec{\mathbf{a}}_1| = |\vec{\mathbf{a}}_2| = a$.

$$F_T - f_1 = m_1 a\,, \qquad f_1 = \mu_k N_1 = \mu_k m_1 g \quad \rightarrow \quad F_T = m_1 (a + \mu_k g)\,.$$

$$F - F_T - f_1 - f_2 = m_2 a \quad \rightarrow \quad F = (m_1 + m_2) a + \mu_k g (3m_1 + m_2)\,.$$

2. A block of mass m on a horizontal surface is attached to the end of a massless spring with stiffness constant k. The other end of the spring is fixed. The mass is given an initial displacement x_0 from equilibrium, and an initial speed v_0 as illustrated in the figure. The gravitational acceleration is g. Ignore friction.



(a) (9 Pts.) Find its maximum speed during its motion in terms of the given quantities.

(b) (9 Pts.) Find its maximum stretch from equilibrium in terms of the given quantities.

Now assume that there is friction between the block and the horizontal surface, and that the coefficient of kinetic friction is μ_k.

(c) (7 Pts.) What is the total distance traveled by the block if it comes to rest at the equilibrium position of the spring?

Solution:

(a) Since there is no friction, total mechanical energy is conserved throughout the motion.

$$U_i = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 \,.$$

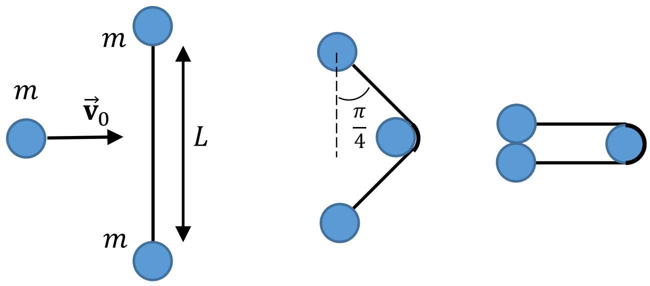
$$U = \frac{1}{2}mv_{\text{max}}^2 \,, \qquad U = U_i \quad \rightarrow \quad \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 \quad \rightarrow \quad v_{\text{max}} = \sqrt{v_0^2 + \frac{k}{m}x_0^2} \,.$$

$$U = \frac{1}{2}kx_{\text{max}}^2 \,, \qquad U = U_i \quad \rightarrow \quad \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 \quad \rightarrow \quad x_{\text{max}} = \sqrt{\frac{m}{k}v_0^2 + x_0^2} \,.$$

(c) Now there is friction, and the block comes to rest at the equilibrium position of the spring. This means $E_f = 0$. Work energy theorem implies

$$W_f = -\mu_k mgD = 0 - \frac{1}{2}mv_0^2 - \frac{1}{2}kx_0^2 \quad \rightarrow \quad D = \frac{1}{2\mu_k mg}(mv_0^2 + kx_0^2) \,.$$

3. Two blocks of equal mass m are tied by an inextensible string of length L, and are at rest on a horizontal frictionless plane. Another identical block, moving with velocity $\vec{\mathbf{v}}_0$ in a direction perpendicular to the string, hits the string at its midpoint. The incoming block drags the two masses, which finally collide and stick together. The string remains taut throughout the process. You can regard all three masses as point masses.



(a) (8 Pts.) What is the final velocity of the three blocks after the collision?

(b) (8 Pts.) What were the speeds of the two dragged masses just before they collide?

(c) (9 Pts.) What was the speed of the dragging mass when the strings made an angle $\pi/4$ with their initial direction?

Solution: (a) Since the string is inextensible, and momentum is conserved during the collision process, all three masses move in the x direction with the same velocity after the collision. Therefore,

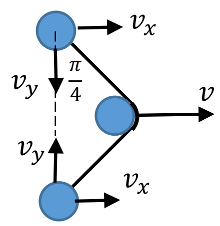
$$m\vec{\mathbf{v}}_0 = 3m\vec{\mathbf{v}}_f \quad \to \quad \vec{\mathbf{v}}_f = \frac{1}{3}\vec{\mathbf{v}}_0 \,.$$

(b) Conservation of momentum in the y direction means the y component of the velocity of the dragged masses are equal in magnitude v_y but opposite in direction. Furthermore, until the two dragged masses collide kinetic energy of the system remains constant. Therefore, just before the collision we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{v_0}{3}\right)^2 + 2\left\{\frac{1}{2}m\left[\left(\frac{v_0}{3}\right)^2 + v_y^2\right]\right\} \quad \to \quad v_y = \frac{v_0}{\sqrt{3}}.$$

So, speed is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{v_0}{3}\right)^2 + \left(\frac{v_0}{\sqrt{3}}\right)^2} \quad \to \quad v = \frac{2}{3}v_0 \,.$$



(c) The figure shows the situation at that instant. The velocity of the dragging mass is in the x direction, denoted by v, and the two dragged masses have x and y velocity components denoted by v_x and v_x, as shown. Since both energy and momentum is conserved, we have

$$mv_0 = mv + 2mv_x \quad \to \quad v + 2v_x = v_0 \quad \to \quad v_x = \frac{v_0 - v}{2},$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + 2\left[\frac{1}{2}m\left(v_x^2 + v_y^2\right)\right] \quad \to \quad v^2 + 2v_x^2 + 2v_y^2 = v_0^2 \,.$$

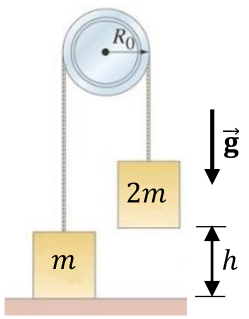
Since the string is inextensible, velocity of the dragged masses relative to the dragging mass must be perpendicular to the string. This means x and y components of the relative veocity must be equal (because of the $\pi/4$ angle).

$$v_{x-\mathrm{rel}} = v_x - v\,, \qquad v_{y-\mathrm{rel}} = -v_y \quad \to \quad v_x + v_y - v = 0 \quad \to \quad v_y = \frac{3v - v_0}{2}.$$

$$v^2 + 2\left(\frac{v_0 - v}{2}\right)^2 + 2\left(\frac{3v - v_0}{2}\right)^2 = v_0^2 \quad \to \quad v = \frac{2}{3}v_0 \,.$$

So speed of the dragging mass is $v = 2v_0/3$.

4. Two masses, m and $2m$ are connected by a rope that hangs over a pulley as shown in the figure. The pulley has radius R_0, moment of inertia I, and is mounted on frictionless bearings. Initially m is on the ground and $2m$ is at rest a distance h above the ground. The gravitational acceleration is g. Assume that the rope does not slip on the pulley when the system is released.



(a) (7 Pts.) Use conservation of energy to determine the speed of $2m$ just before it strikes the ground.

(b) (8 Pts.) What is the angular acceleration of the pulley during the motion?

(c) (10 Pts.) Now assume that there is a constant frictional torque on the pulley. If the $2m$ is given a downward initial speed v_0 and stops just before hitting the ground after moving down a distance h, what is the magintude of the frictional torque?

Solution:

(a)

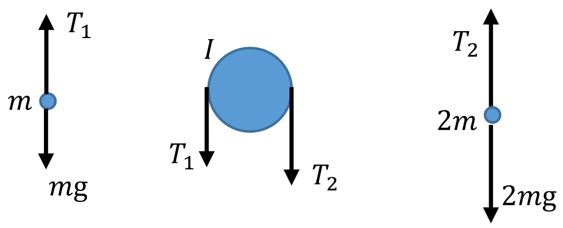
$$E_i = 2mgh\,, \qquad E_f = mgh + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}I\left(\frac{v}{R_0}\right)^2, \qquad E_f = E_i \quad \rightarrow \quad v = \sqrt{\frac{2mgh}{3m + I/R_0^2}}\,.$$

(b) From free body diagrams, we have

$$T_1 - mg = m\alpha R_0,$$

$$(T_2 - T_1)R_0 = I\alpha,$$

$$2mg - T_2 = 2m\alpha R_0,$$



where we have used $a = \alpha R_0$. (The rope does not slip on the pulley.) Solving these equations for α, we get

$$\alpha = \frac{mgR_0}{3mR_0^2 + I} = \frac{1}{R_0}\left(\frac{mg}{3m + I/R_0^2}\right).$$

Or:

$$at_f = v\,, \qquad \frac{1}{2}at_f^2 = h \quad \rightarrow \quad a = \frac{v^2}{2a}\,, \qquad \alpha = \frac{a}{R_0} = \frac{v^2}{2R_0 h} = \frac{1}{R_0}\left(\frac{mg}{3m + I/R_0^2}\right).$$

(c) Let τ denote the frictional torque. As the masses move the distance h the pulley rotates through the angle h/R_0. Work done by friction is equal to the change in the total mechanical energy of the system. We have

$$E_i = \frac{1}{2}mv_0^2 + \frac{1}{2}(2m)v_0^2 + \frac{1}{2}I\omega^2 + 2mgh\,, \qquad E_f = mgh\,, \qquad W_f = -\tau\left(\frac{h}{R_0}\right) = E_f - E_i\,.$$

So

$$\tau\left(\frac{h}{R_0}\right) = \frac{1}{2}(3m)v_0^2 + \frac{1}{2}I\left(\frac{v_0}{R_0}\right)^2 + mgh \quad \rightarrow \quad \tau = \frac{R_0}{h}\left[\frac{1}{2}\left(3m + \frac{I}{R_0^2}\right)v_0^2 + mgh\right].$$